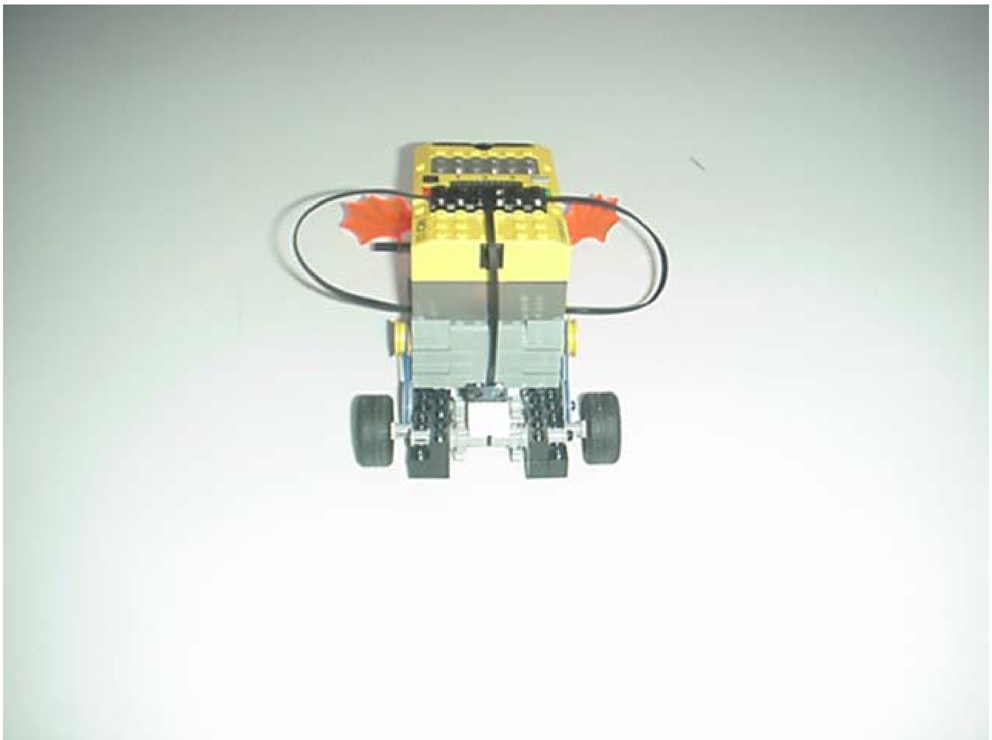
Driving Math



Using Mindstorms for Schools
in a math class at business college

Introduction

The idea arose during a seminar entitled ”Science and Ensuring Welfare” that was held by LEGO Educational Division and Mikroværkstedet. At the seminar, a variety of participants from the educational sector gathered to discuss how to get young people interested in science.

Part of the seminar involved the participants building a little LEGO robot that could be programmed via an infrared connection to a computer. We were divided into groups of four, and the group members did not know each other. Our task was to build a robot that could remove some LEGO bricks from a marked-off area on a table as quickly as possible. We were not allowed to touch the robot or allow it to drive off the edge of the table. We were given about an hour to build and program the robot, and we did not know the programming language or what we would be asked to build beforehand.

The different groups then competed to come up with the best solution. As it turned out, the task required both a lot of collaboration and a variety of skills, which resulted in a great creative and highly focused atmosphere. We were given just enough time to complete the task.

The different groups actually came up with very different solutions—each with its own advantages and disadvantages. As an educational experience, the idea was to hit just the right level of difficulty, so that the task was not so difficult that it created anxiety, but not so easy that the participants were bored.

It was this “event-oriented” pedagogical approach—involving a concrete goal, time pressure, collaboration and the resulting active engagement—that I decided to try in one of my math classes at the business college where I teach.

Preparation

The idea was to introduce the topic of algorithms to a second-year college math class, and to illustrate the topic by using LEGO robots. The class was required to work with an elective topic at some point, and this turned out to be algorithms. With only thirteen students, the class was small, so the equipment needed to carry out the project was not very expensive.

The class quickly agreed to the idea, and I managed to persuade the administration to invest in a Mindstorms for Schools Starter Set, an Enhancement Set, two extra motors and 4 rotation sensors. We ordered these products from Mikroværkstedet in Odense, Denmark, and they arrived the next day, the Friday before the start of fall vacation.

On the Monday after fall vacation, the school's systems administrator placed four computers in the library, installed Robolab on the computers and connected the infrared transmitter. A copy of the teacher's instructions was provided for each computer, thus making four complete workstations. Since there were thirteen students in the class, we could have up to four groups each working with its own robot. Each group could have its own computer and its own RCX programmable brick. However, two groups had to share the bricks in one set. We finally settled on dividing the class into three workgroups with four to five students in each. Tuesday morning, I worked with a computer instructor to build a little robot that could drive, and we tested some simple programs that controlled the robot's movement. Later that same Tuesday, the class started work on the first assignment. The "One Meter" project.

Lectures

Lectures covering the theory behind the project were given on a regular basis throughout the duration of the project, which took 30-35 class sessions.

These lectures dealt with the definition of algorithms, examples of algorithms used, for example, to divide polynomials, to find roots and to determine whether a number is a primary number, and different types of instructions (sequence instructions, selection instructions and iteration instructions) in algorithms. We then went on to learn about the special icon-based programming language used in the Robolab software.

One project requirement was that, during the course of the four assignments, all the groups were to create robot programs that, combined, contained all three types of instructions.

First project: "One Meter"

The first assignment involved building and programming a robot that could drive a distance of exactly one meter. Once the robot started moving, it could not be controlled externally, but only by the program it contained. A meter was measured and marked out with black tape on the floor, and a time limit of four and a half class sessions was set. Within this time, the three groups had to test programs and model ideas as often as the class time allowed. A competition was held at the end of the project, and the group whose robot came closest to the goal won a prize. A journalist from the local newspaper was invited to attend the competition to take pictures and conduct an interview. All this made the project more "event-oriented," and heightened the sense of challenge.

One of the three groups consisted solely of women, and this group started by using a class session to sort the bricks contained in the set and put them in the small compartments, as well as checking to make sure that the set contained the stated number of different bricks. The groups consisting of male students started right away, leaving all the bricks in a big disorganized pile.

One of these groups spent some time programming a victory tune that the robot would play as it crossed the finish line. Clearly, this group expected to win the competition. Yet it was the women's group that won. The journalist from the newspaper was also a woman, and she was very enthusiastic about the success of the women's team.

Programming ideas for the "One Meter" assignment

In general, the students based their work on one of two programming ideas.

1) Distance equal to speed multiplied by time. The robot program establishes a power setting (the power determines the speed of the motor) and the running time for one motor. The time setting was adjusted following a test drive at a set motor power using the trial and error method. One of the groups used a rubber band to transfer power from the motor to the wheels, but they ran into trouble because the tension of the rubber band pulled the robot back from the finishing line after it had stopped. In addition to this, problems sometimes arose when the batteries ran down, causing the motor to run more slowly at any given power setting.

2) The length of track covered is proportional to the number of revolutions of the robot's (front) wheels. Rotations sensors were placed on the front axle, and the software was programmed to perform an iteration instruction (a loop) that repeated one rotation of the wheel. The program allows you to define how many times the loop should run. One problem was caused by a false assumption: the belief that one program loop could be repeated 7.5 times and that this would cause 7.5 revolutions of the wheel.

The math behind the "One Meter" assignment

In addition to the use of algorithms, the math behind the "One Meter" assignment includes the linear connection "direct proportionality." The track distance traveled is $s = vt$, where v represents the speed in meters per second and s is measured in meters.

If the rotation sensor is used, it is possible to involve the concept of radians. A degree of rotation of x radians is equivalent to an arc length and thus a track length of xr, where the radius of the wheel (the circle) is r. When the number of revolutions is registered, the traveled track length is $s = O\omega$, where ω represents the number of revolutions and $O = 2\pi r$ represents the outside circumference of the wheel.

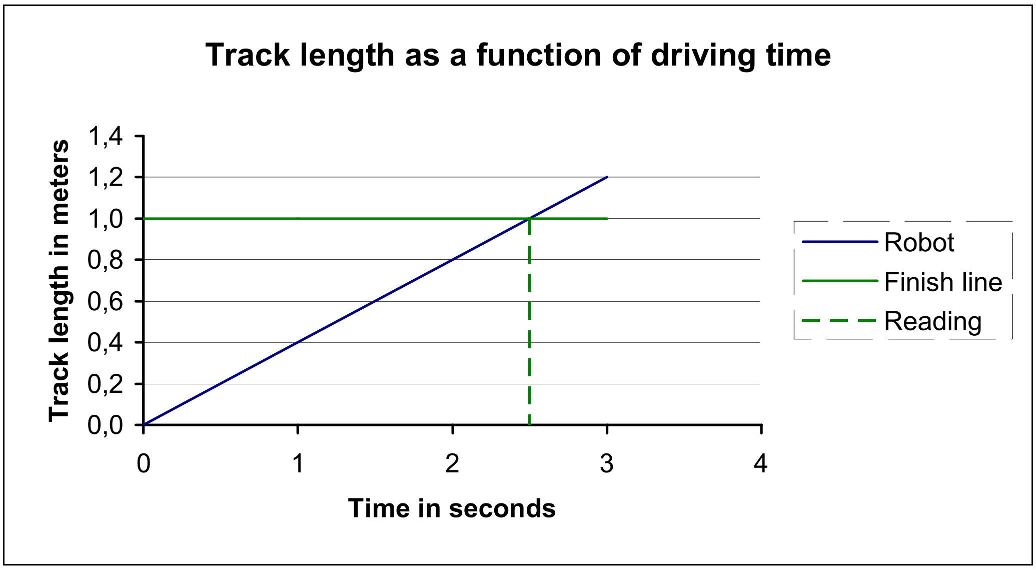


Fig. 1. The relationship between the track length and the driving time. The graph is a straight line through point (0,0)

The required time can be determined if the speed is known. A greater motor power corresponds to a higher speed. By performing a 1-second (or 1-revolution) test drive, you can determine the value of *v* (or *O*), which corresponds to the gradient of the line in figure 1. However, in this kind of investigation a certain amount of downtime can occur before the car actually starts moving. At higher speeds, the line shows a steeper gradient, and a shorter time can suffice. In the Robolab software, the power—and thus also the speed—can be adjusted (almost) infinitely. The same goes for the time. The driving time (or the number of revolutions) setting required to drive precisely one meter can be found by solving the equation $vt = 1$ to find the value of *t* (or the equation $O\omega = 1$ to find ω).

In practice, the students tried different solutions without necessarily being aware of their thought process. If the assignment were changed so that the students were not informed of the required track length until just before the competition started, and no test drive were allowed after announcing the required distance, the students might be forced to be more aware of the mathematical theory involved.

Written assignment for the "One Meter" project

Write about your group's solution to the "One Meter" assignment, describing both the design of the solution and the programming involved. Explain why your algorithm worked or did not work.

Second project: "Five Points on a Parabola"

The second assignment involved constructing and programming a robot that could drive along a continuous track on a horizontal driving surface, passing five predetermined points. Once the robot started moving, it was not allowed to be controlled externally, but only by the program it contained. A series of coordinates was marked off in blue tape on a piece of floorboard. There were axis marks each 0.2 meters, and five points were marked with X's, see fig. 2. The five points lie on the parabola with the equation $y = 5x^2$. Units are meters.

A time limit of about five and a half class sessions was set. The final deadline was set in the middle of the project. Within this time, the three groups of students had to test programs and model ideas as often as the class time allowed.

1) A competition was held at the set time, and the group whose robot came closest to the goal won a prize. In order to check whether the robot actually passed the points, empty plastic bottles (1/2 liter soda bottles) were placed on the points. To prove that the robots had passed the points, they had to push the bottles away. If a robot did not pass all the points, it had to pass as many as possible.

The group whose robot passed the most points in the shortest time won. As it turned out, none of the groups passed all five points. The winning group passed four.

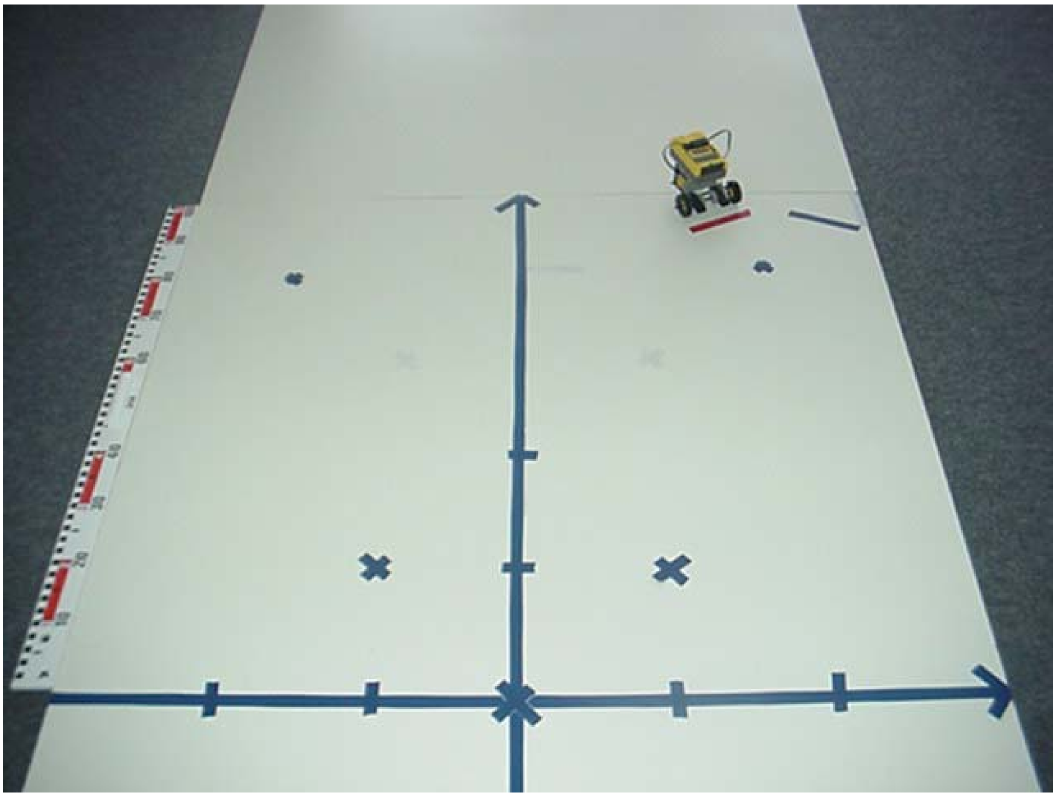


Fig. 2. Floor board with a series of coordinates and five points on a parabola $\boldsymbol{y = 5x^2}$

Programming ideas for the "Five points on a parabola" assignment

My idea was that the students should make the robot move along an arc by using two motors, one on the right wheel and one on the left. If the motors run at different speeds, the robot will travel around a circle with a radius defined by the difference between the speed of the motors and the size of the wheels. By keeping the relative power of the motors adjusted, it should be possible to make the robot follow a course that comes close to following the parabola with the equation $y = 5x^2$. The parabola was thus to be created by creating a series of arcs.

The students used a different idea, programming the robot to follow segments of a straight line and then turning near to or directly over the marked points. They used two motors to make the robot turn, as I had envisioned, programming the motors on the right and left wheels to run at different speeds and even in opposite directions for a certain amount of time. The robot's programs were adjusted after test drives on the course.

The students also had to spend some time working on the vehicles, since running into the bottles required a sturdy construction, and controlling the wheel rotation demanded a very stable transfer of power to the wheels. One group built a robot with caterpillar tracks to achieve more stable and precise movement. The groups also had to know where on the track the robots were to start. However, all the groups chose to start near the point (0,4; 0,8).

Ideas for the math behind the "Five points on a parable" assignment

If a robot has two wheels with motors running at different speeds represented by $f = \frac{v_{indre}}{v_{ydre}}$, the robot will drive in a circle with the (outer-wheel) radius $r = \frac{d}{1-f}$, where *d* represents the distance between the two wheels.

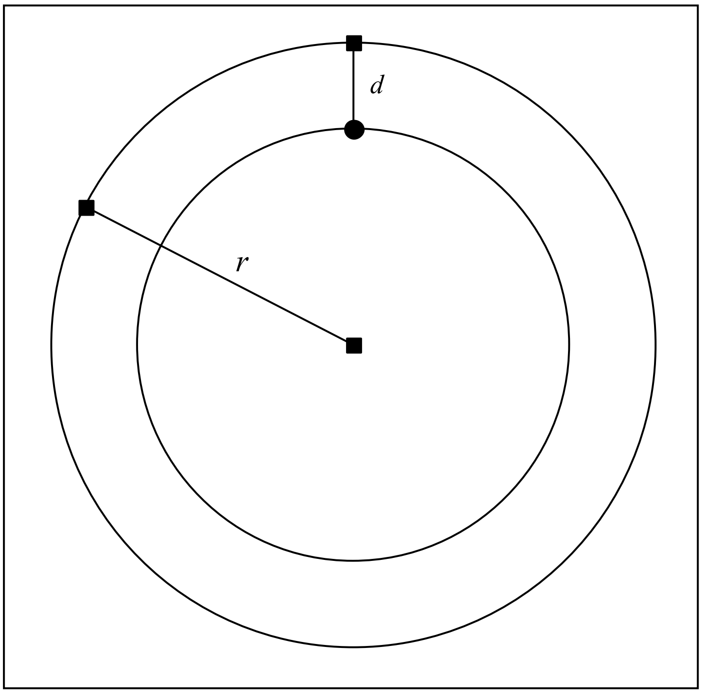


Fig. 3. Tracks of a robot driven by two motors running at different speeds

Resolution of the formula $\boldsymbol{r = \frac{d}{1-f}}$ (1)

The outside wheel runs at the speed $v_{outside}$ and the inside wheel at v_{inside}. We call the time taken to travel once around the circle *T*. The stretch of track traveled is $v_{outside}T = 2\pi r$ in the outer diameter, while the track traveled is $v_{inside}T = 2\pi(r-d)$ in the inner diameter.
Through division we arrive at

$$\frac{v_{inside}T}{v_{outside}T} = \frac{2\pi(r-d)}{2\pi r}$$

$\Leftrightarrow$

$$\frac{v_{inside}}{v_{outside}} = \frac{(r-d)}{r}$$

We use the letter *f* to denote the relationship between the speeds of the inner and outer wheels, and note that *f* must be less than 1, if the distance between the wheels is greater than zero.

$$\frac{v_{inside}}{v_{outside}} = \frac{(r-d)}{r} = f < 1 \quad (2)$$

The last equality in (2) means that

$$r - d = rf$$

$\Leftrightarrow$

$$r - rf = d$$

$\Leftrightarrow$

$$r(1-f) = d$$

$\Leftrightarrow$

$$r = \frac{d}{1-f}$$

An investigation of the correspondence between the power of the motors and the speed of a test robot was carried out (by the teacher) by measuring the distance traveled in one second as a function of the motor power. The result is probably not reproducible, since the speed at a given motor power also depends on the level of the batteries. The results are shown in fig. 4. It turned out that the relationship between power x and speed $y = h(x)$ is best described using a logarithmic model with the formula $h(x) = 0.24 + 0.1113\ln(x)$, where *ln* is the natural logarithmic function.

In principle, the relationship between the motor speeds should be able to be stated as a function of the combined power of the inside and the outside motors. Let us use the letters x and y to represent the power of the inside and outside motors. The relationship between the speeds will then be a function of two variables, x and y:

$$f = \frac{0.24 + 0.1113\ln(x)}{0.24 + 0.1113\ln(y)} \quad (3)$$

This means that the fraction $\frac{1}{1-f}$ will also be a function of x and y. We call this function $g(x, y)$

$$g(x, y) = \frac{1}{1-f} \quad (4)$$

and by substituting in equation (1) we arrive at the following expression for the radius of the track of the outer wheel:

$$r = d \cdot g(x, y) \quad (5)$$

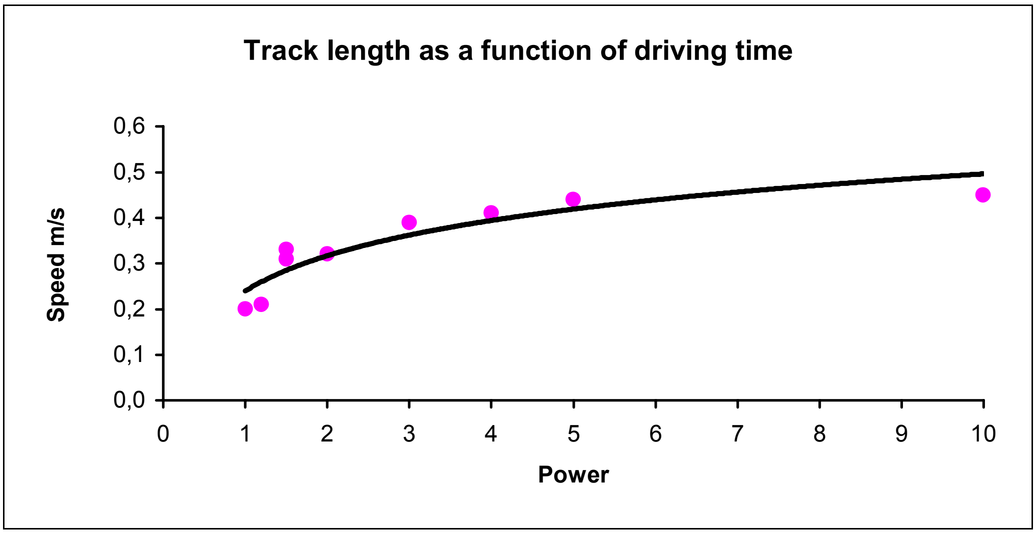


Fig. 4. The relationship between the power of the motors and the speed of a test robot. The solid line of regression can be represented by the equation y = 0.24 + 0.1113ln(x)

By putting the values of the function $g(x, y)$ into a matrix for selected values of x and y with the actual distance between the wheels, it is possible to understand and so plan which circular track the robot should follow. The size of the angle of the arc to be traveled can by governed by setting the driving time t, since the fraction of a full circle to be traveled is indicated by $\frac{t}{T}$ (the arc measured in degrees is proportional to t).

The idea was that sections 1 and 3 on the parabola (see fig. 5) between points (0,4; 0,8) and (0,2;0,2) and (-0,4; 0,8) and (-0,2;0,2) should be achieved by approximating a circle with a large radius, while section 2 between points (0,2; 0,2) and (-0,2;0,2) should be achieved on a course of an arc with a small radius.

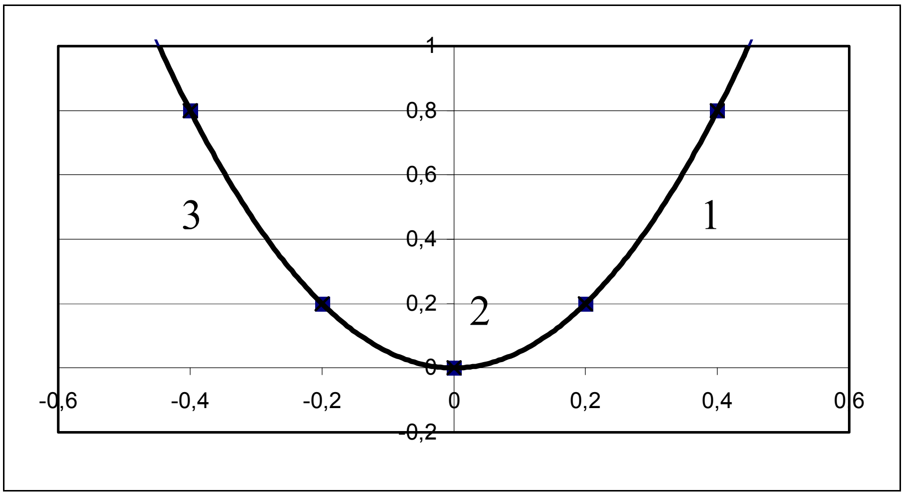


Fig. 5. The parabola $y = 5x^2$

Written assignment for the "Five points on a parabola" project

Describe your solution to the Mindstorms for Schools assignment "Five Points on a Parabola", and use a combination of mathematical formulas, explained in ordinary speech explain why you expected the algorithm and robot you chose to use to solve the task. Provide a critique of your solution and suggest improvements.

Third project: "Twenty Centimeters Upwards"

The third assignment consisted of building a robot that could drive up a slope and stop when it had moved precisely 20 cm, measured vertically from the starting height. Each group was also given a thin wooden stick of their own choice, with a cross section of approx. 3 x 4 mm and a length of either 48 cm or 25 cm (cut from a paint stirrer). The stick was to supplement the LEGO bricks in the Mindstorms for Schools sets.

Within a given period of time, the three groups were asked to test programs and model ideas as often as the class time allowed. A competition was held at the set time, and the group whose robot came closest to the goal won a prize. However, the angle of the gradient and the time at which the competition was to be held was not announced (by the teacher) until ten minutes beforehand, and the students were only allowed a single test drive on the competition gradient. The idea behind keeping the angle of the gradient secret (which made this assignment more difficult than the previous two) was that the students should be forced to carry out more theoretical planning, instead of simply finding a trial and error solution.

Programming ideas for the "Twenty Centimeters Upwards" assignment

The sticks that the students were given fit into some of the cylindrical LEGO elements. This allows the students to use these as "sensors." Combined with an axle linked to a rotation sensor, this meant it was possible for the vehicle to register when it had reached the desired angle and corresponding height. Some geometrical calculations were necessary in order to program the robot so that it used a rotation sensor to determine where to stop. The smallest change that the rotation sensor is able to detect is 1/16 of a revolution, corresponding to 22.5 degrees. For this reason, one of the groups experimented with placing a gear between the measuring stick and the rotation sensor to improve the precision of the angle measurements.

One group based their work on another idea, using the stick as a vertical measuring device linked to a "trailer" that drive in parallel to the vehicle on the slope, but on the horizontal floor. The yellow LEGO element called a "gearbox" was built into the robot and served as a guide for the measuring stick. A black line was drawn across the stick at a height of exactly 20 cm. This line was detected by a light sensor, which triggered the robot to stop. This solution required less use of geometry, but more imagination and practical work in designing the robot.

Ideas for the math behind the "Twenty Centimeters Upwards" assignment

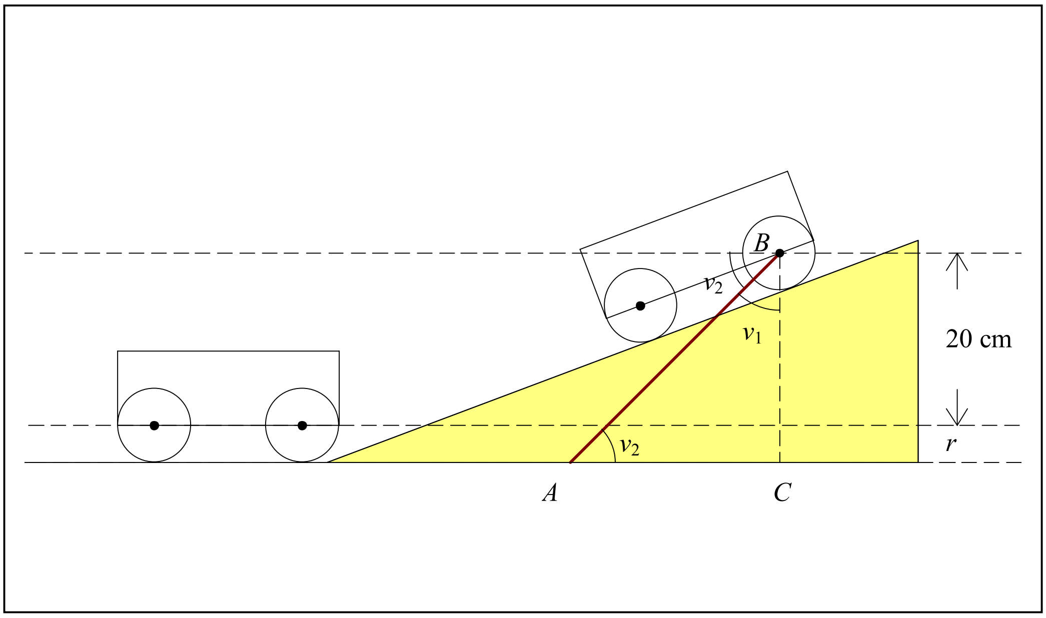


Fig. 5. Sketch of a robot on its way up a slope in the "Twenty Centimeters Upwards" project. The robot starts on the horizontal base in front of the slope. The line *AB* illustrates the stick, and the angle *BCA* is a right angle. The radius of the wheels is *r*

A wooden stick, *AB* (see fig. 5), is attached to a rotation sensor that measures the rotation of the stick around point *B*. If the stick is horizontal at the start, it will move at an angle of v_2 during the robot's ascent of the slope. This angle appears again as angle *A* in triangle *ABC*, since *AC* is also horizontal and thus parallel to

the horizontal dotted line through *B* (see fig. 5). Triangle *ABC* is a right-angled triangle with *C* as the right angle.
For triangle *ABC*, $|AB| = 48$ cm and $|BC| = 20 + r$, where *r* represents the distance between the base and the axis of rotation that the rotation sensor uses to detect angles (in this case, the radius of the robot's wheels).

Triangle *ABC* produces

$$\sin v_2 = \frac{20 + r}{|AB|} \quad (5)$$

and when r and |AB| are known, angle v_2 can be calculated through the equation (5), since we know that $v_2 < 90^\circ$:

$$v_2 = \sin^{-1}\left(\frac{20 + r}{|AB|}\right)$$

The task now is to program the robot so that the value of the rotation sensor is set to zero at the start while the stick is held in a horizontal position, and the robot stops when the rotation sensor has reached the pre-calculated value v_2. Since the rotation sensor only measures in steps of 22.5 degrees, it's a good idea to use a (toothed) gear between the stick and the rotation sensor, so that a small rotation angle on the stick becomes a large rotation angle for the rotation sensor. Here, the number of teeth in the gear must be included in calculating the value of the angle/number of rotations detected by the rotation sensor that will make the robot stop.

Written assignment for the "Twenty Centimeters Upwards" project

Describe your solution to the Mindstorms for Schools assignment "Twenty Centimeters Upwards", and use a combination of mathematical formulas and plain speech to explain why you expected the algorithm and robot you chose to use to solve the task. Provide a critique of your solution and suggest improvements.

Fourth project: "Stop at the Top"

The fourth assignment involved constructing a robot that could drive across an unfamiliar landscape and stop on top of the first or second hill (see fig. 6).

The hilly landscape was created using a long (23 % 240 cm) pliable plate (chip board) covered with abrasive paper to ensure sufficient friction. Within a given period of time, the three groups had to test programs and model ideas on a model course as often as the class time allowed.

At a set time, wooden blocks were used to create the driving surface into the actual landscape. The groups were only allowed to perform a single test drive on the final course before the competition was held, where the group whose robot stopped closest to the hilltop won a prize.

If two groups were equally close to the top, the group that had chosen to stop on top of the second hill won, since the programming involved in making the robot stop on the second hill was considered to be more difficult than that required to make the robot stop on the first hill.

The project started with a written assignment designed to ensure that the students considered some math and practical aspects, to avoid resorting to on-the-spot ideas and trial and error.

Written assignment for the "Stop on the Top" project

Outline a robotic vehicle and Robolab program algorithm that solves the problem of how to make a robot drive straight across an unknown hilly landscape and stop at the top of the first hill. (Difficult version: Stop at the very top of the second hill).

You must include
1) A sketch of the robot
2) An algorithm
3) An explanation of why your vehicle and algorithm will solve the problem

Ideas for the math behind the "Stop at the Top" assignment

The idea behind this assignment was that the students should recall or discover knowledge pertaining to all hilltops, regardless of their particular shape. In this case, the relevant knowledge is the fact that there is a horizontal tangent with a maximum on the graph for a differential function. A differential function has a cohesive and smooth projection, and when the hilly landscape is shaped from a single surface one can reasonably assume that the contours of the landscape can be roughly described as the graph for a function that can be differentiated.

From a mathematical perspective, the line between two points where the robot's wheels touch the hilly landscape will correspond to a secant on the contours of the landscape, and when the robot is on the top of the hill, the slope of this secant can also be assumed to be close to zero.

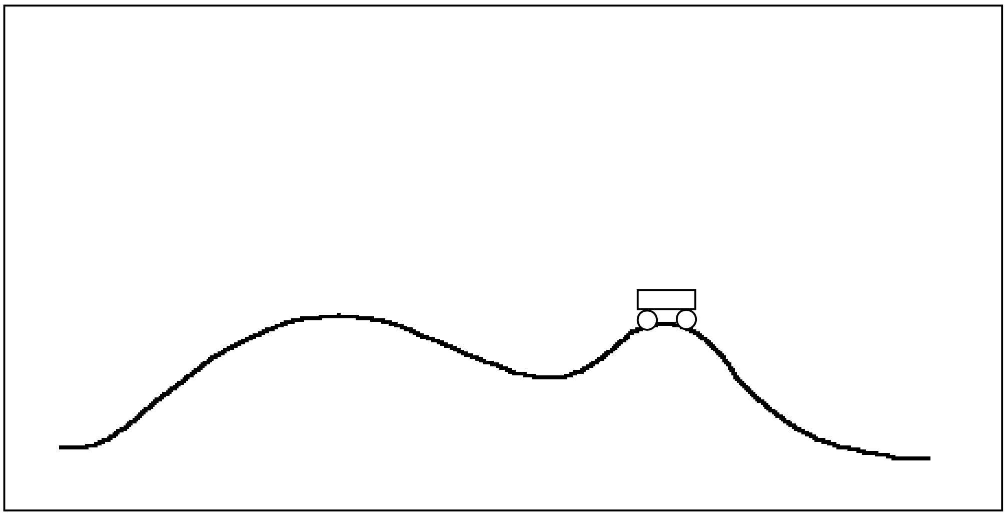


Fig. 6. Hilly landscape, where the robot must stop on top of the second hill

Programming ideas for the "Stop at the Top" assignment

The students' models and programs were based on the idea of detecting when the robot was positioned horizontally. They equipped their robots with different types of pendulums or similar devices that rested vertically when the robot reached a horizontal position. The position of the pendulum was read by a device such as a rotation or light sensor.

Pitfalls included forgetting to set the sensor to zero at the start of the course and the fact that oscillations of the pendulum could cause the robot to stop prematurely. The robot can be made to stop on the second hilltop by adding a loop (iteration instruction) to the programming. A pitfall in this connection is that the robot is also in a horizontal position in the valley between the two hills, which means that the loop must be repeated three times before the robot stops.

The atmosphere in class

The atmosphere in class was active and distinguished by a sense of challenge, strategic planning, surprises, commitment, creativity, a focus on results and intense concentration. The students were waiting outside the door before I arrived, just waiting for class to start. Sometimes they asked if we really had to stop at the end of the day.

Conclusion

It is possible to create a learning situation that generates a sense of challenge and active engagement. This experiment only touched on part of what Mindstorms for Schools can do, and proved that experimentation is an important aspect of teaching math.

However, teachers might find it difficult to accommodate such experiments with the formal curriculum. In this case, the requirements were met by running the project as an elective course in algorithms. It could also be organized as an interdisciplinary project between math and computer science, or as a project designed to practice required topics such as linear functions, trigonometry and conical sections. Finally, the project could be included as an elective course in parameter curves.

Other possible options could include a counter with lights that count in binary, measuring reaction times, a robot that can keep track of its own speed and adjust it to maintain an constant level, etc. It might also be possible to design assignments which incorporate even more provocative challenges that make yet higher demands on abstract thinking.

Appendix 1

Student materials on algorithms

All students were given:

2) A copy of pages 61-64 of the teacher's guide to Mindstorms for Schools. These pages contain a list of the icons used in programming the RCX.
3) A copy of pages 249-251, the Algorithms section in chapter 8, "Rekursion og induktion", from the book "Matematik 3", Carstensen and Frandsen, Systime 1990.
4) A copy of pages 317-320 (using algorithms to determine whether a number is a primary number) from chapter 10, "Talteori", in the book "Matematik 3", Carstensen and Frandsen, Systime 1990.
5) A copy of pages 341-346 of chapter 11, "Ligninger och algoritmer", from the book "Matematik 3", Carstensen and Frandsen, Systime 1990.
6) A copy of pages 281-298 of chapter 16, "Algoritmer", from the book "Elementær Matematik", Henrik Grell and Bent Willum Hansen, Sigmat 1990.